BIBLIOGRAPHY

ON

THE POLEMIC PROBLEM.

WHAT IS THE VALUE OF

π

"Diruit, ædificat, mutat quadrāta rotundis."—HORACE.

COMPILED BY

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The Extension of the Decimals of π.

The extension of the decimals of the orthodox value of π is credited to several mathematicians as follows:

Peter Metius, (15— -1636) . .	6	places.
Francis Vieta, (1540-1603) . . .	10	"
Adrianus Romanus, . . .	16	"
Cornelius Petrus, ...	32	"
Ludolph Van Ceulen, . . .	36	"
Abraham Sharp, (1651-1742) . .	72	"
James Machin, (16— -1751) . .	100	"
Thomas F. de Lagny, (16— -1734) .	128	"
(Radcliffe Library, Manuscript, Oxford,) .	155	"
Clausen and Dase, of Germany, independently,	200	"
William Rutherford, 1843, . .	441	"
William Shanks, 1853, . . .	607	"
William Shanks, 1873, . . .	707	"

707 Decimals.

π=3.

141592	653589	793238	462643	383279	502884	197169	399375
105820	974944	592307	816406	286208	998628	034825	342117
067982	148086	513282	306647	093844	609550	582231	725359
408128	481117	450284	102701	938521	105559	644622	948954
930381	964428	810975	665933	446128	475648	233786	783165
271201	909145	648566	923460	348610	454326	648213	393607
260249	141273	724587	006606	315588	174881	520920	962829
254091	715364	367892	590360	011330	530548	820466	521384
146951	941511	609433	057270	365759	591953	092186	117381
932611	793105	118548	074462	379834	749567	351885	752724
891227	938183	011949	129833	673362	441936	643086	021395
016092	448077	230943	628553	096620	275569	397986	950222
474996	206074	970304	123668	861995	110089	202383	770213
141694	119029	885825	446816	397999	046597	000817	002963
123773	813420	841307	914511	839805	70985±		

Constants.

Common logarithm of $\pi = 0.4971498726941338543 5+$

Naperian logarithm of $\pi = 1.1447298858494001741 4+$

Reciprocal of $\pi = 0.318309886183790671537767526745 +$

Square of $\pi = 9.869604401089358618834490999876 +$

Square root of $\pi = 1.772453850905516027298167483341 +$

Naperian base, $\varepsilon = 2.718281828459045235360287471352 +$

Common logarithm of $\varepsilon = 0.434294481903251827651128918916 +$

Naperian logarithm of $\varepsilon = 1.000000000000000000000000000000 +$

$$\sqrt{\varepsilon^{\pi}} = 4.81047748 1+$$

○ □ —

Cyclometry — Quadrature — Rectification.

—○=□—

Cyclometry is the science of *circle-measuring;* a quadrature is the *making of a square* equivalent to a given circle; rectification of the circle is the *finding of a straight line* equal to the circumference of a given circle. These problems are one and the same in the sequel, and have engaged the attention of geometers from the earliest ages. The object of this paper is not to discuss the various methods devised to solve the famous problems, but to give a brief account of some of the quite numerous productions on the subject, and this in answer to many inquiries from all parts of the world, as to what has been written on the perplexing problems.

The secret key to the problems is the true value of the ratio of the diameter to the circumference of any circle, which ratio is denominated by the Greek letter π (*pi*), the initial of the word *periphereia*, the circumference of a circle.

The first recorded instances of a value of π are found in the Bible. One used by King Solomon in the making of vessels for the Temple:

> "And he made a molten sea, ten cubits from one brim to the other: *it was* round all about, and his height *was* five cubits; and a line of thirty cubits did compass it round about."
> [*I Kings* VII, 23.]

> "Also he made a molten sea of ten cubits from brim to brim, round in compass, and five cubits the height thereof; and a line of thirty cubits did compass it round about."
> [*II Chron.* II, 2.]

The translation of Julia E. Smith gives these texts slightly different, the latter of which is as follows:

> "And he will make the molten sea ten by the cubit from its lip to its lip, rounded round about; and five by the cubit its height; and a cord thirty by the cubit will surround it round about."

King Solomon's ratio (3) can be explained only that he measured the diameter from the outside, and the circumference on the inside, of the cord encircling the top of the molten sea.

There is another obscure allusion to a value of π found in the name *Eliezer* (which in Hebrew numerals is 318), the steward of Abram's house (*Gen.* XV, 2); this is a circumference value to a diameter 100. Eliezer was the "instructor" of *three hundred and eighteen* "trained *ser-*

vants" (*Gen.* XIV, 13). Josephus (*Ant.* Bk. I, VIII, 2) says Abram "communicated arithmetic and delivered the science of astronomy to the Egyptians." See J. R. Skinner's "Sources of Measure," p. 208, for more information on this value of π.

The earliest work giving an account of the many attempts to square a circle is that by J. E. Montucla, entitled *Histoire des Recherches sur la Quadrature du Cercle*, Paris, 1754. He adds to the title, "A Book intended to make known the Real Discoveries concerning this Celebrated Problem, and to serve as a Preventative against new attempts at its Solution." How far he has succeeded will appear in the following pages. From a translation of this work we mention some of the earlier attempts to discover the quadrature.

Archimedes, about 250 B. C., applied himself to the problem and showed that the value of π was less than $3\frac{10}{70}$ and more than $3\frac{10}{71}$. Campanus, author of the work "Tetragonismus," published in 1503, one of the earliest of two *printed* book on the quadrature, claims that the ratio of Archimedes was exactly $3\frac{1}{7}$, or $3.142857\frac{1}{7}$. Archimedes' work was entitled *De Dimensione Circuli.*

Aristotle mentions two of his contemporaries, Antiphon and Bryson, who worked on the problem. Antiphon's method was to find the area of the circle by adding to the inscribed square the area of four isosceles triangles in the four segments, also the sum the eight similar triangles in the remaining segments, and so on till the circle was exhausted. Bryson's method resulted in the ratio, $3\frac{3}{4}$, or 3.75 !

Sextus, a disciple of Pythagoras, claimed to have solved it, but his method has not come down to us. Aristophanes, in his "Comedy of the Clouds," ridicules Meton of Metonic-Cycle fame, for endeavoring to find the valve of π.

Nicomedes and Appolonius made researches on the problem ; the former by means of the curve called *the quadratrix*, the discovery of which has been ascribed to Dinostratos. Eutocious tells us that Appolonius had approached nearer the true ratio than Archimedes did. Philo of Gadara had approached still nearer, so that his ratio differed by less than $\frac{1}{100000}$ from the usually accepted ratio. Anaxagoras, while in prison, spent much time on the problem. Hypocrates of Chios while searching for the ratio was led to the discovery of the exact area of the *lune* or cresent-shaped figure, which can be demonstrated to be exactly equivalent to a given square.

Cardinal Nicholas de Cusa rolled a wheel on a plane, and then he measured the path of one revolution of the wheel and made the ratio to be the $\sqrt{10} = 3.1622776+$. He also conceived of the curve made by a point in the rim of the wheel passing through space, called the cycloid, and believed with Charles Bovillus, in the next century, that it was the arc of a circle.

Oronce Finée, a Royal professor in 1544 published a quadrature a little more ingenious than Bovillus; Monantheuli, another Royal professor, in 1600, published his ratio. In 1592, Joseph Scaliger published his *Nova Cyclometria*, giving the ratio, 3.14098+; being shown his error by five geometer ,he would not surrender. The only quadrator on record who, it is said, was convinced of his error, and acknowledged the same, was Richard White (Albinus), a Jesuit; his book is called *Chrysespis sen Quadratura Circuli.*

Montucla mentions many others who have spent much time and labor to discover the value of π, bringing the history of the subject down to the publication of his work.

Montucla says, speaking of France, that he finds three objects prevalent among cyclometers:

1. That there was a large reward offered for the solution
2. That the longitude problem depended upon the solution.
3. That the great end and object of geometry depended upon it.

In 1872, a work was published in London, entitled " A Budget of Paradoxes," by Prof. Augustus De Morgan, of Trinity College, Cambridge. It is composed of the collected articles, correspondence, reviews of books, etc., by Mr. De Morgan, published in the London *Athenæum* from 1863 to 1870. In this work of 512 pages there are mentioned the names of 75 writers on the subject of "Cyclometry." Mr. De Morgan has reviewed the works of 42 of these writers, giving the results of their search for the value of π, bringing the subject down to 1870. The entire list has been compiled and tabulated by the writer of this monograph, which will accompany this paper. An examination of the compilation does not reveal the name of a single American author or book on the subject of "Cyclometry."

During the past six years as editor of the American *Miscellaneous Notes and Queries*, we have received many kinds of questions involving the value of π, and such questions have been discarded from the magazine on account of the endless discussion they engender. But of late the literature on "Cyclometry" having been inquired for, we at once decided to publish a bibliography and brief review of such works as our library furnished and a few mentioned in our serial literature.

Of the 100 titles given in this bibliography, 52 are bound volumes, 32 are pamphlets, 7 are broadsides, and the remaining 9, including one manuscript, are communications to the press. These books have not been collected as a specialty, but are what naturally find their way on a variety of subjects into a mathematical collection of 700 or 800 volumes, and 500 or 600 pamphlets on "the bewitching science" of mathematics.

Those who desire to investigate these works, and the ingenious methods proposed to find the value of π, can fully satisfy themselves that there are many roads to Rome. Many of the works are elaborate, and accompanied with artistic plates, and ample diagrams.

Chambers' Encyclopædia, *Article*, "Quadrature of the Circle," says: "If an equation could be discovered for $\sqrt{a} + \sqrt{b}$, a and b representing irrational quantities, it would be welcomed as the solution of the grand problem."

Theodore Faber proposed the following equation as that desideratum: $\sqrt{a}+\sqrt{b} = \sqrt{a + b + \sqrt{4ab}}.$ The result is however an irrational quantity as the area of a circle and equal to a parallelogram and convertible into a square by the usual rules, but not a *square* by his New Law in Geometry. That square will forever lack one square unit, however infinitesimal the measure-unit may be assumed.

Dr. Charles Hutton says, in writing on this subject, that he divides the writers on this problem into two classes: The first, consisting of able geometricians not led away by illusions, are those who seek only for the approximation more and more exact, whose researches have often terminated in discoveries in almost every part of geometry. Second, those who are less acquainted with the principles of geometry and try to solve the problem by analogies and paralogisms.

However this may be, the results of many of them greatly differ, and that too among some able geometers. We think the object in view is to find a *finite* ratio which shall be the *true* value of π. All admit that the ratio should be finite. The results of 63 of the writers in the following bibliography are given, tabulated and classified at the end, for comparison.

Bibliography — Cyclometry and Quadratures.

ADORNO, JUAN NEPOMUCENO. Introduction to the Harmony of the Universe; or Principles of Physico-Harmonic Geometry. Plato said, "The Great Geometrician is God." The harmony of the universe proves the truth of this sublime sentence. Royal 8vo, cloth, pp. 160. 72 elaborate diagrams. London, 1851.

This is a very elaborate work on harmony, proportion, analogy, and ratio. The author says that he is convinced that "the circumference of any circle to its diameter is precisely as 22 to 7, a proportion considered by Archimedes as an approximation only." His ratio corresponds with that of William A. Myers, $3.142857\frac{1}{7}$

ANGHERA, DOMENICO, REV. Quadratura del Cerchio. 8vo. cloth. Malta, 1858.

This priest says: "The circle is four times the square inscribed in its semicircle." Hence his area is .80, and his ratio is $3\frac{1}{5}$, or 3.2.

BADDELEY, WILLIAM. Mechanical Quadrature of the Circle- — *London Mechanics' Magazine*, August, 1833.

"From a piece of carefully rolled sheet brass was cut out a circle $1\frac{9}{10}$ inches diameter, and a square $1\frac{7}{10}$ inches diameter. On weighing them they were found to be of exactly the same weight, which proves that, as each are of the same thickness, the surfaces must also be precisely similar. The rule, therefore, is that the square is to the circle as 17 to 19."

Hence this would make his ratio, $3.202216\frac{24}{361}$; area, $.800554\frac{6}{361}$.

ANONYMOUS. Resumé for Analytic Exercise. 4to. Construction:

To determine the point towards which an infinite descending series of triangles tend to a final term, being the limit of that segment of spiral which is the evolute of the quarter-circle.

To show the relations of lines representing the third root of quantities which are to each other as one and two, and the angle of the radius with the spiral which results from constructing the equation of the "two mean proportionals."

To transfer to any portion of the arc the conditions for its division into the same proportional parts as those of the semi-circle divided by the radius.

The author presents diagrams of circles and triangles combined, and shows that the radius of one circle is "$AB = \frac{1}{3}\pi \div 4$," (which, if we understand him rightly,)$=.26179387+$, the ratio, $3.1415926535+$.

BENNETT, JOHN. Original Geometrical Illustrations ; or the Book of Lines, Square, Circles, Triangles, Polygons, &c., showing an easy and scientific analysis for increasing, decreasing, and altering any given circle, square, triangle, ellipse, parallelogram, polygon, &c., to any other figure containing the same area, by plain and simple methods, laid down agreeably to mathematical demonstrations; intended as a complete instructor to the most useful science of Geometry and Mensuration. 4to. cloth, pp. text, 70; plates, 54. Frontispiece, a diagram—The Circle, Square, and Triangle—primitive geometrical figures. London, 1837.

[Second Book.] The Arcanum, Comprising a Concise Theory of Practical Elementary and Definite Geometry ; exhibiting the Various Transmutations of Superfices and Solids ; obtaining also their Actual Capacity by the Mathematical Scale ; including Solutions to the yet Unanswered Problems of the Ancients—The Circle, Square, and Rectangle of Similar Areas. 8vo. cloth. pp. 48. 176 diagrams. Frontispiece, a diagram—The Problem of Napoleon Buonaparte to his Staff, resolved and drawn by John Bennett. London, 1838.

Mr. Bennett says that the problem of corresponding areas of the square and circle " has remained altogether in obscurity ; although, rewards were offered by Charles V, of 1000 crowns ; and the States of Holland a similar sum, to any person effecting it ; but it does not appear to ever have been performed." He quadrates the circle thus : " The transverse of the circle being divided into 26 equal parts, 21 of those parts are found to occupy one-fourth of the circumference." Then he constructs the equi-areal square by intersecting the circumference at the 8 points of the 84 parts in the circumference, leaving 12 parts without the circumference and then 9 parts within the circumference, alternately. This is a mechanical quadrature, and give for a ratio, $\frac{84}{26}$, or $3.230769\frac{3}{13}$; area, $.807692\frac{16}{169}$, which is not in accord with his elaborate and artistic diagrams throughout his works.

BENSON, LAWRENCE SLUTER. Scientific Disquisitions concerning the Circle and Ellipse ; a Discussion of the Properties of the Straight Line and the Curve, with a critical examination of the Algebraic Analysis. " If a better *system's* thine, impart it frankly, or make use of mine." 12mo. cloth, pp. 94. Aiken, S. C., 1862.

Prof. Benson has published some twenty pamphlets, more or less on the area of the circle, three volumes of philosophic essays, and one geometry—" The Elements of Euclid and Legendre, Excluding the *Reductio ad Absurdum* Reasoning." He endeavors to demonstrate that the area of the circle is equal to $3R^2$, or the arithmetical square between the inscribed and circumscribed squares. His theorem is: "The $\sqrt{12} = 3.4641016+$ is the ratio between the diameter of a circle and the perimeter of its equivalent square." The ratio between the diameter and circumference, he believes, is not a function of the area of the circle. He accepts the value of $\pi = 3.141592+$; but the area of the ◯, he believes, is ⇔ $3R^2$, or .75.

BROWER, WILLIAM., M. D. The Quadrature of the Circle; being a full Exposition of the Problem. 8vo. pamphlet, pp. 16. 4 plates. Philadelphia, 1874.

The entire pamphlet is devoted to geometrical constructions and algebraic equations. According to his demonstration, he says:

"The circle is equal to the inscribed square, $+2\frac{1}{2}$(side of inscribed square) $\times$ (width of quadrantal segment), $+2$(side of inscribed octagon) $\times$ (width of octagonal segment), -2(width of octagonal segment) $\times$ ($\frac{1}{2}$side of inscribed square$-\frac{1}{2}$side of inscribed octagon$-$width of quadrantal segment)."

Dr. Brower is ingenious and goes through many demonstrations, but the two triangles which he calls analogues are not similar, as he supposes, and therein lies his error. His first trial for the ratio results in $3.152075+$, which he finds greater than the accepted ratio, and he concludes that a certain segment is less than x, an unknown quantity.

BOYAI, JANOS. La science absolute de l'espace. 8vo. Paris, 1868. We have never seen this work.

BOUCHÉ, CHARLES P. The Regulated Area of the Circle, and the Area of the Surface of the Sphere. 8vo. pamphlet, pp. 64. Cincinnati, Ohio, 1854.

Mr. Bouché says in the year 1823 he fixed the ratio to be $3\frac{13}{81}$, or $3.160493827\frac{13}{81}$; but "in 1833 he found himself constrained to correct it to $3.1684+$; and later he found himself compelled to correct this." He finally made the ratio, $3.17124864+$. The first ratio ($3\frac{13}{81}$) was first developed by M. de Fauré in his "Dissertation, Découverte, et Démonstrations de la Quadrature Mathématique du Cercle," Geneva, 1747; and "Analyse de la Quadrature du Cercle," Hague, 1749, mentioned by De Morgan in his "Budget of Paradoxes," p. 89. This same ratio was developed by Theodore Faber in 1865.

B., G. W. Squaring the Circle ; the Exact Circumference. — *Manufacturer and Builder*, Vol. III, No., 2, p. 31, February, 1871,

Mr. G's constructed diagram, and proposition amounts to this :

" When we draw an equilateral triangle, of which each side is equal to the diameter of a given circle, one-fourth of the circumference will be equal to the radius plus one-third of the perpendicular of this triangle."

This proposition gives for the ratio, 3.1547005+, a number larger than a circumscribed polygon of 48 sides.

Carrick, Alick. The Secret of the Circle ; its Area Ascertained. Second edition. 8vo. pp. 48. London, 1876.

Mr. Carrick, with the help of ten diagrams, some colored, concludes that the ratio is $3\frac{1}{7}$, or $3.142857\frac{1}{7}$; and its area, $.785714\frac{2}{7}$. He ends his essay with these words ; *Patet omnibus veritas, multam ex illâ etiam futuris relicta est.*

Cart, Francis Guerin. The Problem of Centuries ; What is the True Relation of Circumference to Diameter ? Diagrams.—*News and Courier*, (Charleston, S. C.) August 2, 1876.

Mr. Cart denies the universal correctness of the " 47th of Euclid." He took out a copyright of his new discoveries, September 14, 1875. under the title, " New light on an old subject, or an analysis of the present received science of geometry, showing its errors and revealing the truth." His proposition is as follows :

" The area of the circle is equal to the area of its circumscribed square minus the area of a rectangle whose hight is one-half the radius and whose side is the altitude of an equilateral triangle having the diameter for its base."

Hence his area of circle is $1-\frac{1}{4}\sqrt{75}$, or .78349364+ ; this makes the ratio, 3.13397456+. He deduces it from $3\frac{209}{1560}$, or 3 1339745+.

Carter, R. Kelso Capt. The Quadrature of the Circle ; an Answer to Prof. Lawrence S. Benson's Proof that the Area of the Circle is Equal to Three Times the Square of the Radius. 8vo. pamphlet, pp. 16. Chester, Pa., 1876.

Capt. Carter says, " Prof. Benson's method of proof is so ingeniously conducted that a very close study is necessary to discover its fallacy.' Prof. Benson believes that the area of a circle is equal to $3R^2$, or .75, and that the area is not a function of the ratio.

CHASE, PLINY EARLE., LLD. Approximate Quadrature of the Circle. Published June 16, 1879. Haverford, Penn.

"On the rectangular co-ordinates X, Y, lay off, from a scale of equal parts, $AB=3$, $AC=20$, $AX=60$, $AD=9$. Join DC, and draw DE parallel to DC. Take $EY=AC$, and join XY. Then

XY : AC : : circumference : diameter, *nearly*."

This gives the ratio, 3.14158499+.

CLARYVANCE. J. Geometrical Approximations of the Quadrature of of the Circle. 8vo. London, 1852.

We have never seen this work neither a notice nor review, only a catalogue announcement.

CRABB, NORMAN. Geometrical Square Root; a Circle Quadrated, and other problems. 16mo. pamphlet, pp. 29. Chicago, Ill., 1879.

Mr. Crabb says the "rule he has adopted for quadrating a circle has never been published or taught." He gives twelve problems and ten diagrams in his little work and finds the ratio to be $3\frac{1}{7}$, or $3.142857\frac{1}{7}$. This is the same ratio that William A. Myers, and Juan Nepomuceno Adorno arrived at in their elaborate treatises.

CRAIGE, JOHN. Methodus Figurarum Lineis, Rectis and Curvis, comprehensarum Quadratus determindi. 4to. pp. 43. London. 1685.

This work seems to be a discussion of quadratures in general.

DAVIES, CHARLES, LL.D. An Examination of the Demonstrations of Davies' Legendre, showing how the Polygon becomes a Circle, by the Methods of Newton. 12mo. pamphlet, pp. 36, New York, 1873.

Prof. Davies reviews his own edition of Legendre, elucidating various methods by the principles of the Calculus, and makes them the foundations of mathematical science. He demonstrates the usually accepted ratio, 3.141592+, sometimes designated "the orthodox ratio."

DAVIS, JOHN. The Measure of the Circle. The Use and Importance of the Measure, discovered in January, 1845. 8vo. cloth, pp 156. Providence, R. I., 1854.

Mr. Davis says: "In confidence, I have found the point; to find the circumference of any circle, great or small: multiply the diameter by $9\frac{5}{10}$, and divide the product by 3; this gives you the perfect circumference, in all cases." Hence his ratio is $\frac{19}{6}$, or $3.166666\frac{2}{3}$.

DEMEDICI, CHARLES. The New Science—Mathematical Commensuration. 12mo. cloth, pp. 196. 50 diagrams. Chicago, 1883.

DEMEDICI, CHARLES. The Medician Theorem ; a Scientific Exposition of the Geometric Paradox ; founded on newly discovered facts. 4to. pp. 4, chart. New York, 1885.

The Theorem—" Sides of inscribes squares are to sides of circumscribed squares, as sides of any squares are to the diagonals of the same square : and the number $\frac{24}{17}$ expresses finitely the exact ratio common to the sides and the diagonals of any square."

Mr. DeMedici's work is a full exposition of the subject. He quotes Bernoulli's proposition :

" If the number 4 be divided by 1, 5, 9, 13, and every fourth number in succession, and afterwards by 3, 7, 11, 15, and every fourth number thereafter, the difference between the sum of the first set of quotients and that of the second is equal to the ratio of diameter to circumference." Ratio, $\frac{908}{289}$, or $3\frac{41}{289}$, or $3.1418685\frac{35}{289}$.

DINGLE, EDWARD. Balance of Physics. Square of the Circle, and the Earth's True Solar and Lunar Distances discovered and demonstrated, as by astronomical facts in the Eclipses.—" The secret of the Lord *is* with them that fear him, and it is for them to know."—PSALM XXV, 14. 8vo. cloth, pp. 246. London, 1885.

This work, like J. N. Adorno's and Wm. A. Myers's, is very elaborate in its computations applied to the universe in all its ramifications. His demonstrations are that the ratio is $3\frac{1}{7}$, or $3.142857\frac{1}{7}$.

DIRCKS, HENRY, LLD. Chimeras of Science : Astrology, Alchemy, Squaring the Circle, Perpetuum Mobile, etc. 12mo. cloth, pp. 48. 6 plates. London, 1869.

He says, refering to Arago, that " the area of the space included within a circle of thirty-eight millions of leagues radius, may be determined with such a degree of precision that the probable error shall not exceed the space of a mite." He presents James Smith's diagram and demonstration of the ratio, $3\frac{1}{8}$, or 3.125, claiming to be a simple and exact method and sufficiently demonstrative.

DRACH, S. M. On the Circle-Area, and Heptagon-Chord. 8vo. Plate. " From 3 diameters deduct 8-thousandths and 7-millionths of a diameter ; to the result and five per cent. The ratio, 3.14156265+."

" DURHAM," (N. H.) How to Square the Circle.—*The Sun*, (N. Y.), August, 1878.

Proposition—"Let it be assumed that the area of the circle is equal to the excesses of four squares over four circles described within the squares whose diameters and sides are equal. If this proposition can be geometrically demonstrated, the area of a circle can be deduced as a corollary from said demonstration, for it makes the area of five circles equal to the area of four squares, and consequently the area of a circle equal to four-fifths of the square of its own diameter."

Hence his area is .80, and his ratio, 3.2.

FABER, THEODORE. Mathematical and Philosophical Manifesto, declaring numerous theorems, problems, postulates, corollaries, axioms, propositions, rules, and facts, hitherto unknown in science, and naturally growing out of the Extraordinary and Most Significant Discovery of a Lacking Link in the demonstration of the world-renowned Pythagorean Problem, utterly disproving its absolute truth, although demonstrated as such for twenty-three centuries; and by this discovery establishing the fact of the Perfect Harmony between Arithmetic and Geometry as a Law of Nature ; and calculated to settle forever the famous dispute between the two Great Philosophic Schools. 8vo. pamphlet, pp. 34. New York, 1872.

Mr. Faber published his "New Mathematical System" first in 1865. In 1879, he published his third pamphlet, "A New Law in Geometry, leading to the Solution of Unsolved Problems." "An eternal geometrical difference between a square and a so-called irrational quantity." He denies the universality of the "47th problem of Euclid," the only few cases of its exact application being special cases or coincidences; that a circumscribed square is not equal to two inscribed *squares*. He demonstrates the ratio to be $3\frac{13}{81}$, or $3.160493827\frac{13}{81}$, the same as that of M. de Fauré, in 1747, and that of Charles P. Bouché, in 1823, which after ten years he abandoned. Mr. Faber's ratio, $3\frac{13}{81}$, is a square whose root is $1\frac{7}{9}$, which root also is a square whose root is $\frac{4}{3}$, or $1\frac{1}{3}$. His area of the circle, $.790123456\frac{64}{81}$, is a square whose root is $.8888\frac{8}{9}$. Circumscribed square being 1; inscribed square $=(.7\frac{1}{14})^2-\frac{1}{196}$.

FERREL, WILLIAM. Converging Series expressing the Ratio between Diameter and the Circumference. 4to. pp. 6. (Smithsonian Contributions to Knowledge, No. 233). Washington, 1871.

He says the paper is "a method for obtaining converging series expressing the value of π, and the series obtained, are thought to be new." The result produced is the accepted value of the ratio, 3.141592+.

FLEMING, PETER. Geometrical Solutions of the Quadrature of the Circle. Large 4to. cloth, pp. 10. 6 plates. Montreal, 1850.

Mr. Fleming says : "When the length of the circumference of a given circle can be found, or resolved into a straight line, the quadrature of the circle is accomplished. It is now the solution of this problem which the writer presumes to lay before the public."

FLEMING, PETER. Geometrical Solutions of the Length and Division of Circular Arcs; the Quadrature of the Circle, Trisection of an Angle, Duplication of the Cube, and the Quadrature of the Hyperbola. 8vo. cloth, pp. 40. 5 plates. Montreal, 1851.

This is a new edition of the former work elaborated with additions.

"FINALITY, A." The Circle Squared. — *Courier*, Boston, Mass., January 28, 1872.

This cyclometer says that "the arc of each six equal segments gains $\frac{1}{18}$ of each chord in length; therefore, the proportion is $\frac{19}{6}$, and hence his ratio is $3.16666\frac{2}{3}$." Area, $.791666\frac{2}{3}$.

FISHER, THOMAS. Mathematics Simplified and made Attractive; or the Laws of Motion Explained. 8vo. cloth, pp. 128. 19 plates. Philadelphia, 1853.

Mr. Fisher devotes pages 56-76 to the subject of the circle. He says no other method has been devised, and he believes never will be, that the area of a circle can be obtained, than that by the "method of exhaustions." He concludes that the ratio is the usual orthodox value, $3.141592+$. He closes his book with a poem of sixteen stanzas on the "The Creation of Light"

"FUTURUS." The Quadrature of the Circle; a Puzzle for Mathematicians. Glasgow journal, May, 1853.

Construction of diagram: "Make ABC a right angle, angle at B. Make $AB = \frac{1}{6}$diameter, $BC = \frac{1}{3}$diameter, $BD = \frac{1}{5}$diameter, $BE = \frac{1}{7}$diameter, and $BF = \frac{1}{12}$diameter, (the latter BF on perpendicular.) Join AC, AD, AE, and draw through F a straight line cutting AE at H and meeting AC at G, so that $HG = ED$. Then $2BC + FG = \pi \times$diameter$\div 4$, whence may be derived the square."

Therefore, his ratio is $3.1238093\frac{11}{21}$, and the area, $.7809523\frac{8}{21}$.

GEE, WILLIAM F. A New System of Geometry successfully applied to the Solution of the Square of the Circle; published as a Supplement to the Geometry of the present time. 12mo. cloth, pp. 68. Charleston, S. C., 1859.

Mr. Gee's calculations result in a combination of the commonly received ratio and the decimal of one-seventh. His ratio seems to be $3.14159269777454 2\frac{6}{7}$. Area, $.78539317444363 5\frac{5}{7}$.

Gidney, Charles T. The Ratio of Circumference to the Diameter of the Circle.—*Troy* (N. Y.) *Standard*, October 2, 1878.

Mr. Gidney works by a series of algebraic equations numbering 1350, from which he developes the ratio, 3.135135+.

Goodsell, Samuel C. A Book of Stubborn Facts; appreciating unknown conditions at the Base of Construction of Plane-Figures; the Square the only Regular Polygon of Known Area, and the reason why; proving circumference ÷ diameter to be the consequent of dividing 4 times the least by $\frac{9}{10}$ the greatest diameter of the square; and the circumference of the greatest diameter of the square to be $\frac{10}{9}$ the sum of the four sides. Also, the area of the circle to be circumference^2 $\times$ .08, or diameter^2 $\times$ $\frac{64}{81}$, and solidity of the sphere = diameter^3 $\times$ $\frac{128}{243}$ The ratio of diameter to circumference being a sequent of diameter, line and area, we may safely lay aside the Geometer's Approximate for the more delicate Ratio of Pure Mathematics; Containing valuable tables, such as natural sines, tangents, etc, etc., of improved exactness. Tables of equal line and relative area — of equal area and relative line — of unknown figures. Also, round and square diameters of numbers; area and circumference of circles, solidities of the spheres, etc., etc.; short comprehensive formulæ pertaining to line and area, illustrating the maxim, — "Truth may languish, but never die." 8vo., New Haven, Conn., 1875.

Diameter of □ being 1, the D. = $\sqrt{1 \text{ area}}$, L. = $\sqrt{16 \times \text{area}}$;

R. = $\sqrt{16 \times 1.}$

Diameter of ◯ 1, the D. = $\sqrt{\frac{81}{64} \text{ area}}$; L. = $\sqrt{12\frac{1}{2} \times \text{area}}$;

R. = $\sqrt{12\frac{1}{2} \times \frac{64}{81}}$.

The title-page of the prospective work indicates the results of his formulæ. His ratio is 3.1426968+. Area, .7856742+.

Gould, Lucius D. A Practical and Mathematical Demonstration of Fnding the Circumference, and Squaring the Circle, when the Diameter is given.—From "The Carpenter's and Builders's Assistant and Wood-Worker's Guide." 8vo. pp. 2. New York.

"To find the side of a square, the area of which shall be equal to the area of the circle, divide the diameter of the circle by 14 and multiply the quotient by 11; add to the product $\frac{1}{10}$ of the diameter, and *annex* the first figure of the quotient."

This rule results from the ratio $\frac{22}{7}$, or 3.142857$\frac{1}{7}$. Mr. DeForest P Lozier, of Newark, N. J., also supports this ratio

GRAY, J. H. A General Formula for Inscribed Polygons. — In the *Ohio Educational Journal.*

Prof. Gray's reduction fills a full page, and the last result is this:

$$2^7\times\sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}=128\times.024543=3.1415+.$$

GROSVENOR, CYRUS PITT. The Quadrature of the Circle Perfected, or the Circle Squared; in which the method is stated and demonstrated for determining with perfect accuracy the area of any circle of a given radius, and the length of its circumference; and consequently the length of any arc of the circumference; which is done according to geometrical principles, and not on the schemes hitherto employed by mathematicians, by which only approximation to the truth has ever been accomplished. Together with rules for practical mensuration of curvilinear figures, both plain and solid. 4to. pamphlet, pp. 10. Plates and diagrams. New York, 1868.

His rule for the area is stated as follows:

"Square the diameter of any circle, multiply the square by two, extract the square root of the product, from the root subtract the diameter of the circle, square the remainder, multiply this square by five-fourths, subtract the product from the square of the diameter of the circle."

Rev. Mr. Grosvenor's ratio is 3.14213562373090506S+, and the area, .78553390593272626 7+.

HARBORD, H. The Circle Squared, From *Hull and Eastern Counties Herald*, (England), February 27, 1868.

Mr. Harbord's calculations are to find the *finite* value of π. His results are as follows:

Ratio:	3.14159265358938193239974916.
Square root of ratio:	1.7724538509054.
Side of a square:	.8862269254527.
Area of circle:	.785398163397365483099937 29.

HART, DAVID S. Quadrature of the Circle. The sum of the infinite series, $\frac{8}{1\times3}+\frac{8}{5\times7}+\frac{8}{9+11}+\frac{8}{1.\times15}+\frac{8}{17\times19}$ &c., *ad infinitum*, if it can be found, will solve the problem. Proposed in the *Yates County Chronicle.*

James Smith says this series calculated to 50 terms makes the ratio less than 3 133+, and to whatever number of terms carried the sum can never be made to reach 3.14.

HARRIS, JOHN. (Kuklos.) The Circle and Straight Line. Parts I, II, III. Plates. 3 Volumes, and Supplement. 8vos. cloth, pp. 42, 56, 26, 26. Montreal, 1874.

These volumes are elegantly executed mechanically, the plates being in separate volumes. He demonstrates the ratio to be 3.142696+. Area, .785674+.

HERSCHEL, A. S. (Collingwood, Eng.) On an Approximate and Graphical Rectification of the Circle. — *The Mathematical Monthly*, Vol. III, No. 5, pp. 152-155, February, 1861. (From the London *Quarterly Journal of Pure and Applied Mathematics*, October, 1868.

The value of the ratio is founded on the singularly close relation that is made by the angle $Tan^{-1}(\frac{1}{4}\pi)$ to a root of the equation $Sec\ x = Cot\ x$. This results in the form $Cos\ x = Tan\ x = \sqrt{\left\{\frac{\sqrt{(5)}-1}{2}\right\}} = .7863+$. While $\frac{1}{4}\pi = .7853+$.

HILL, THOMAS. An Elementary Treatise on Curvature; also, a Fragmentary Essay on Curves. 8vo. pamphlet, pp. 30. Boston and Cambridge, 1850.

Prof. Hill's treatise is a specialty on curves discussing first principles of curves. He deduces the usually accepted ratio, 3.141592+ The essay is exampled by the problem found in Gill's *Mathematical Miscellany*, May, 1839, p. 43, and solved by Prof. Benjamin Peirce, the proposer: " Find a curve which is its own evolute." The involute and evolute of the circle are correlate, and the evolute of any algebraic curve is rectifiable.

HORNISH, J. K., President and General Manager of the Vulcan Smelting and Mining Company, Denver, Col., 1885. MS. 4to.

Mr. Hornish's Ratio is $3\frac{5}{32}$, or 3.15625. We are not aware that he has yet published his demonstration.

HOBBES, THOMAS, of Malmsbury. *Decameron Physilogium*; or, The Dialogues on Natural Philosophy; to which is added the Proportion of a Straight Line to Half the Arc of a Quadrant. 12mo. pamphlet, pp. 136. London, 1678.

The author says of his proposition " for the domonstration whereof, we must assume certain known Truths, and Dictates of common Sense." His value of the ratio is $\sqrt{10} = 3.1622777+$.

HOULSTON, WILLIAM. The Circle Secerned from the Square, and its Area Gauged in terms of a Triangle common to Both ; also, an Original, Simple and Exact Method pointed out approximating as closely as possible to the numerical value of the Triangle, and the consequent veritable Content of the Inscribed Circle, in Relation to any given Square. 4to. pamphlet. pp. 22. London, 1862.

Mr. Houlston's treatise is unique, being interspersed with seventy-four quotations from the poets of all past ages. He makes the ratio, 3.14213562373+. Area, .78553390593+.

HUDSON, W. H. New and Demonstrative Solution of the Geometric Quadrature of the Circle and the Geometric Mean. Indian Chronological Tables. Portrait. 8vo. 3 plates. Calcutta, 1831.

JACKMAN, ALONZO, LL.D. The Circle Squared. 12mo. pamphlet, pp. 8. Northfield, Vt., 1876.

Prof. Jackman of the Norwich University, also published these geometric l demonstrations in quarto form first in 1872, second in 1873 ; hence this pamphlet is the third edition. He substantiates the commonly received ratio, 3.1415926535+.

JACKSON, EDWIN W. A Geometric System for the Measurement of The Area of a Circle, or any of its Sectors. 8vo. pp. 22. 21 plates. New Brunswick, 1826.

He demonstrates that "the circle is conceived to be a polygon of an infinite number of sides and equal to a triangle, the base being equal to the periphery, and its altitude to the radius, therefore, the square, when thrown into an angle of this description, will give the periphery."

MAY, JOHN. The Theory and Construction of the Quadrature of the Circle ; also the Globe or Ball reduced to the Cube, and two New Measures—The Octans, with the Inclination of the Perpendicular Line. 8vo. pamphlet, pp. 20. 4 plates. Philadelphia, 1866.

Mr. May finds the ratio to be $3\frac{6}{25}$, or 3.24! Area of circle=.81.

MERCERON, D. S. The Square Root of Surds ; The Solution of the XLVIIth Problem of Euclid, and Square of Circle, with True Method of Finding the Circumference. 8vo. pp. 13. Baltimore, 1848.

Mr. Merceron's rule is : "Multiply the chord of the arc of 90° by 5, and divide the product by 4, and the quotient will be the circular square-root, which, when multiplied (squared) will give the square area of the circle." He makes the ratio to be $3\frac{1}{8}$, or 3.125. Area of circle, .78125. This ratio is the same as that of James Smith who had an elaborate discussion with several of the leading mathematicians of Europe. Mr. Smith published this correspondence in five octavo volumes, a total of 1988 pages, the correspondence beginning in 1859 and ending in 1872, a period of 13 years. He was reviewed at length by De Morgan in his "Budget of Paradoxes."

Morton, James. The System of Calculating Diameter, Circumference, Area, and Squaring the Circle; together with interest, miscellaneous tables, and other information. 12mo. cloth, pp. 144 Philadelphia, 1879.

Mr. Morton says that it is not his purpose to introduce to the public any new principle, but "the result of laborious calculations culminating in the final elucidation of facts." He finds, therefore, the ratio to be $\frac{402.1238596594935 67+}{128}$ or 3.141592653589793+.

Murdock, W. D. C. A Demonstration of the Quadrature of the Circle. pp. 8. Without date or place of publication. Announced in *The Mathematical Monthly*, Vol. III, No. 11, p. 356, August, 1861.

We have never seen a copy of this quadrature.

Myers, William Alexander. The Quadrature of the Circle, the Square Root of Two, and the Right-Angled Triangle. First Ed., 1873. "Where is the wise."—1st Cor. 1, 20. "Now the serpent was more subtle than any of the beasts of the field which the Lord God had made."—Gen. III, 1. Second edition. 16 plates. 8vo. cloth, pp. 164. Cincinnati, Ohio, 1874.

The introduction comprises 64 pages giving a history of the problem and attempts at the solution, translated from the French of Montucla, by J. Sabin, Louisville, Ky. The author then proceeds to the "geometrical and final solution of the quadrature of the circle by an entirely new method, together with ample proofs of the same," as he says. He finds the ratio to be $3\frac{1}{7}$, or $3.142857\frac{1}{7}$.

Parker, John A. Quadrature of the Circle, containing Demonstrations of the Errors of Geometers in finding the Approximations in Use; with an appendix, and practical questions on the quadrature, applied to the astronomical circles; to which are added lectures on polar magnetism, and non-existence of projectile forces in nature. 27 plates, and diagram. Second edition. 8vo. cloth, pp. 304. New York, 1874. [First edition, 8vo, cloth, pp. 212. New York, 1851].

Mr. Parker sums up his demonstrations and says, "the true ratio of circumference to diameter of all circles is four times the area of an inscribed circle for a ratio of circumference to the area of the circumscribed square for the ratio of diameter." He finds the ratio to be $\frac{20612}{6561}$, or 3.1415942+. Seba Smith of New York published his "New Elements of Geometry," in 1850, and says in the preface of that work that he is convinced of the truth of Mr. Parker's ratio.

Pierce, George W. Squaring the Circle.—*Advertiser* (Boston, Mass.) January 2, 1883.

Mr. Pierce replies to a statement made in the same paper, December 26, 1882, saying that M. Lindemann had proved that the usually accepted value of π "cannot be the root of any algebraic equation whatever with rational coefficients." This Mr. Pierce denies, and says that

$$\text{“}\pi = 2^{nth} \times \sqrt{2-\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2+\sqrt{2}}}}}}}} \text{ \&c.}$$

to n radicals, when n is equal to infinity, each radical sign covering all that follows, this being the perimeter of an inscribed regular polygon of 2 to the *nth* power sides, this expression being the root of an algebraic equation with rational coefficients."

This expression is the same in result as that of J. H. Gray.

Playfair, John. (Supplement to Elements of Geometry, containing the First Six Books of Euclid.) On the Quadrature of the Circle. 8vo. pp. 164-194. New York, 1854.

A demoustration of the method of inscribed and circumscribed polygons, or method of exhaustions, resulting in the ratio, 3.141592+.

Pratt, Henry F. A., M. D. The Mutual Relations of the Circle, the Square, the Cube, and the Sphere. 8vo. pp. 32. Appendix to "Eccentric and Centric Force. a New Theory of Projection." "All things are double, one *against* another." London, 1862.

Dr. Pratt by many analogies and comparisons finds the ratio to be $3\frac{1}{5}$, or 3.2.

"Quadrator." The Square of the Circle, and the True Ratio of the Diameter to the Circumference.

This is a communication to a newspaper. He says that sexagenary arithmetic is in harmony with revolution, rotation, cycles: and circles. He adds the two extremes of the circle and then takes the geometrical mean, thus: $\sqrt{360+1} = 19$. Then this divided by the hexagonal number, 6, because perfect, gives the ratio, $3.166666\frac{2}{3}$, and the area, $.791666\frac{2}{3}$.

Rollwyn, J. A. S. Elementary Difficulties in Geometry : The Duplication of the Cube ; The Trisection of an Angle ; The Quadrature of the Circle. Chapter XXXIII, of "Astronomy Simplified, for general reading with numerous New Explanations and Discoveries." London, 1871. 8vo. pp. 10.

"The area of a circle is equal to three-fourths of the square of its diameter, or three-fourths of the area of the circumscribed square; and that concurrently, twice the area of the circle is equal to three times the area of the inscribed sqnare."

Rule—Multiply the diameter of the circle by itself and deduct one-fourth of the product ; the remaining quantity is the area of the circle.

Mr. Rollwyn's area is the same as that of Prof. L. S. Benson—that it is the arithmetical square between the inscribed and circumscribed squares.

Rossi, Gaetano, of Catanzaro. Soluzione Esatta, e Regolare de-Difficillissimo Problema della Quadratura del Circolo ; Produzzione Sintetica, ed Analitica. *Hæc qui spernit, id esi Semitas Sapentiæ, ei denuncio non recte philosophandum.*—Boetius. Seconda edizione. Londra, 1805. 8vo. pamphlet, pp. 108. 8 diagrams ; portrait.

The author's demonstrations result in the ratio, $3\frac{1}{5}$, and area, 80.

Scholfield, Nathan. On the Rectification and Quadrature of the Circle. Part Fourth of a Series on Elementary and Higher Geometry, Trigonometry, and Mensuration ; containing many valuable Discoveries and Improvements in Mathematical Science, especially in relation to the Quadrature of the Circle, and some other Curves. 8vo. pp. 108–139. New York, 1845.

Mr. Scholfield's treatise is a learned and searching analysis on the subject of curves, segments, spirals, cycloids, revoloids, etc. He substantiates the orthodox ratio, 3.14159265358979323846264З+.

Skinner, J. Ralston. A Criticism on the Legendre Mode of the Rectification of the Curve of the Circle. 8vo. pamphlet, pp. 22. Cincinnati, 1881.

The author says the orthodox value of *pi* obtained by the Legendre method from the sides of the interior polygons is *numerical*, and nor *geometrical*. The circumference of a circle is a *curve* which finally re-enters on itself and forms the boundary of the circle. The numerical values of the polygons are not indicative of the circle penned up between them. Mr. Skinner's demonstrations substantiate the ratio as found by John A. Parker, namely, $\frac{20612}{6561}$, or 3.1415942+.

SHANKS, WILLIAM. Contributions to Mathematics, comprising chiefly the Rectification of the Circle to 607 Places of Decimals. Royal 8vo. pp. 95. London, 1853.

Mr. Shanks here publishes the value of π to 607 decimal places He gives the value of ε (Naperian base) to 137 decimal places, the value of M (Modulus) to 137 decimal places, and the powers of 2 as far as 2^{721}. He was assisted by Dr. William Rutherford in the verification of the first 441 decimals of π. Since the publication of this work, Mr. Shanks has found errors in the last 14 places of the 607 decimals, as printed in this book, corrected the errors, and then extended the decimals to 707 places, and they are printed by the Royal Society of London, in their Proceedings, Vol. XXI, 1873, as follows:

π=3.	141592	653589	793238	462643	383279	502884	197169	399375
	105820	974944	592307	816406	286208	998628	034825	342117
	067982	148086	513282	306647	093844	609550	582231	725359
	408128	481117	450284	102701	938521	105559	644622	948954
	930381	964428	810975	665933	446128	475648	233786	783165
	271201	909145	648566	923460	348610	454326	648213	393607
	260249	141273	724587	006606	315588	174881	520920	962829
	254091	715364	367892	590360	011330	530548	820466	521384
	146951	941511	609433	057270	365759	591953	092186	117381
	932611	793105	118548	074462	379834	749567	351885	752724
	891227	938183	011949	129833	673362	441936	643086	021395
	016092	448077	230943	628553	096620	275569	397986	950222
	474996	206074	970304	123668	861995	110089	202383	770213
	141694	119029	885825	446816	397999	046597	000817	002963
	123773	813420	841307	914511	839805	70985±		

SMITH, JAMES. Relations of a Circle inscribed in a Square. pp. 6

Commensurable Relations between a Circle and other Geometrical Figures. 1860. pp. 32

Quadrature of the Circle; Correspondence with an "Eminent Mathematician." 1864. pp. 188

Nut to Crack for the Readers of De Morgan's "Budget of Paradoxes." 1863. pp. 72

True Ratio between Diameter and Circumference, Geometrically and Mathematically Demonstrated. 1865. pp. 102

British Association in Jeopardy and Prof. De Morgan in the Pillory without hope of escape. 1866. pp. 96

Quadrature and Rectification of the Circle. 1867. pp. 74

Euclid at Fault, in Theorem, Proposition 8, Book VI; and Theorems, Propositions 12 and 13, Bcok II. 1868. pp. 12

SMITH, JAMES. Geometry of the Circle and Mathematics as applied by Geometers and Mathematicians, shown to be a Mockery, Delusion, and a Snare. 1869. pp. 416
Curiosities of Mathematics, Instruction of Mathematicians. pp. 98
The Ratio between Diameter and Circumference Demonstrated by Angles, and Euclid's Theorem, Proposition 23, Book I, Proved to be Fallacious. 1870. pp. 524
Cyclometry and Circle-Squaring in a Nutshell. 1871. pp. 44
Why is Euclid Unsuitable as a Text-book of Geometry? Theorems, Propositions of Euclid, 8 and 13, Book VI, Proved to be Erroneous, by Heterodox Geometers. 1871. pp. 56
Quadrature and Geometry of the Circle Demonstrated. Portrait. London and Liverpool, 1872. pp. 268

These works are profusely illustrated with plates, diagrams, extracts, and examples. He demonstrates the ratio to be $\frac{25}{8}$, or 3.125. He credits Joseph Lacomme with finding this ratio, in 1836, who is found in De Morgan's list. Mr. Smith's works totalize 1988 pages on this subject.

SMITH, SEBA. New Elements of Geometry. Three Parts. I. The Philosophy of Geometry. II. The Demonstrations in Geometry. III. The Harmonies of Geometry. 8vo. pp. 200. New York, 1850. London edition, pp. 200, 1850.

Mr. Smith examined John A. Parker's manuscript quadrature, became convinced of the truth of it, and published his own Geometry the year previous to the publication of Mr. Parker's "Quadrature of the Circle."

SMOOTH, EPHRAIM. Measuring of Circles; the Proportion which the Diameter bears to the Circumference—*Register of Arts and Sciences*, July 8, 1826. London.

Mr. Smooth illustrates both his and Archimedes' ratio by examples, and claims that the ratio is $3\frac{1}{5}$.

SOMERSET, (Duke of). A Treatise in which the Elementary Properties of the Ellipse are deduced from the Propertise of the Circle, and geometrically considered. Illustrated. 8vo. London. 1843.

STACY, JOSEPH. Squaring the Circle.—*Boston Herald*, April 4, 1874.

Mr. Stacy says he "has no more difficulty in obtaining the ratio than in obtaining the diagonal of a square. The circumference of a circle, as near as can be expressed in so many figures, is 3.152955+; the error is less than 1 in 75,000,000; or it makes a difference of 91 miles in the circumference, or 29 miles in the diameter of the earth."

STEELE, JAMES. Exact Numerical Quadrature of the Circle effected regardless of the Circumference, and the Commensurability of the Diagonal and Side of the Square. 8vo. pp. 75. London, 1881.

The author finds the exact area of a circle in the nonary scale, but just how he does not explain: "The circle is equal to 9 square units nonarily expressed as 10. When the circumscribed square is = 2, the circle is 1.570796+." We do not comprehend this statement.

TAGEN, JOANNEM Nep. Quadratura circuli tandem inventa, et mathematice demonstrata; cum II tabulis. Folded diagrams. 8vo. pp. 75. Cassoviæ, 1832. [His methods are very complex.]

TERRY, CONSTANT. A Problem for the World; the Circle Squared. Eagle Pass, Texas, January 20, 1871. Published in *The Investigator*, Boston, Mass., February 22, 1871.

"I demand the solution of a circle whose area, diameter, and circumference are each perfect squares."

1. Circumference 1, diameter=.3169126500570573503741758013 44.

2. Circumference 1, area=.7922816251426433759354395033 6.

3. Diameter 1, circumference=3.155443620884047221646914261131144918692825740436092019081115722 65625.

4. Area 100, diameter=11.25899906842624.

5. Area 100, circum.=35.52713678800500929355621337890625.

6. Area 100=$\frac{1}{2}$circumference$\times\frac{1}{2}$diameter.

7. Multiply area, when circumference is 1, by area, when diameter is 1, and the product is .0625.

8. Multiply diameter, when circumference is 1, by the circumference, when diameter is 1, and the product is 1.

9. Multiply square of diameter by square of circumference, when area is 100, and the product is 160,000.

Square root of (1) is .562949953421312.
Square root of (2) is .281474976710656.
Square root of (3) is 1.7763568394002504646778106689453125.
Square root of (4) is 3.3554432.
Square root of (5) is 5.9604644775390625.

THOMPSON, G. H. The Discovery of the Quadrature; announced to the world by the Divine Assistance.—In *Coram's Champion*, 1826.

Mr. Thompson's quadrature was the forerunner of that developed by Augustus Young nineteen years later in his first edition of "Rational Analysis," 1845. The prime formula upon which it was based is

$$
\begin{array}{cccccccccccc}
{}_{1} & 1 & \overset{3}{\smile} & {}_{1} & 1 & \overset{1}{\smile} & {}_{1} & 1 & \overset{2}{\smile} & 1 & \overset{6}{\smile} & 1 \\
 & \overline{\quad} & & & \overline{\quad} & & & \overline{\quad} & & \overline{\quad} & & \overline{\quad} \\
 & 1 & & & 2 & & & & {}^{1} & 3 & {}^{1} & 4\ {}^{1}
\end{array}
$$

We do not comprehend this formula as published in *Scientific Tracts and Family Lyceum*, Vol. I, p. 157, by Augustus Young, the champion of Mr. Thompson.

THORNTON, EDWARD. The Circle Squared. 8vo. London, 1868.

Mr. Thornton's quadrature agrees precisely with Lawrence S. Benson's, in making the circle-area, $3R^2$, or .75.

UPTON, WILLIAM, B. A. The Circle Squared. Three famous Problems of Antiquity, Geometrically Solved. The Quadrature of the Circle; Diameter definitely expressed in terms of the Circumference; Circumference equalized by a Right Line. The whole rendered intelligible for arithmeticians as well as for geometers; adapted for the higher classes in schools of both sexes, private students, collegians, &c. "*Mutans quadrata rotundis.*"—HORACE. 8vo. pamphlet, pp. 24. Supplement: The Circle Squared; First — Arithmetical Summary; Second—Geometrical Confirmation. "*Finis coronat opus.*" Plates. pp. 8. London, 1872.

The author demonstrates the orthodox ratio, 3.14159265+, by several methods not found in our text-books.

VANDERWEYDE, PHILIP H., M. D. The Philosopher's Stone: Four Essays, containing the Answer of Positive Science to the Question, What is known at present, about the Quadrature of the Circle? 8vo. pamphlet, pp. 40. New York, 1861.

Dr. Vanderweyde, for many years editor of *The Manufacturer and Builder*, has given in this essay an epitomized account of what the subject is, and then endeavors to answer it. The rectification of the circle is answered by several methods of demonstration, resulting in the ratio 3.1415926535+.

WEATHERBY, J. G. To find a Straight Line equal to the Semi-circumference of the Circle.—*Barnes' Teacher's Monthly*, Vol. I, p. 384, July, 1875.

Mr. Weatherby's geometrical construction and equation makes the semi-circumference of a circle of diameter 60, to be 94.6 (nearly). Hence, the ratio, $3.15333\frac{1}{3}$ (nearly).

YOUNG, AUGUSTUS. Unity of Purpose or Rational Analysis; being an Exposition of the Quadrature of the Circle, and the Law of Gravity. "These are not, perhaps, very attractive speculations; they disturb old and favorite associations; they serve to reduce many cherished traditions, much painfully acquired knowledge, to obsolete lore; but these things are so, and we must accustom ourselves to regard them and their consequences without shrinking." Second edition. 8vo. pamphlet, pp. 36. Burlington, Vt., 1853. [First edition was published in Boston, Mass., 1846. 8vo. cloth, pp. 292].

He says his "purpose is to prove to the satisfaction of the world, that the circumference of the circle, whose diameter is unity or 1, is the third or cube root of 32, and hence that the area is the cube root of .5." He finds the ratio, 3.1748020+, and the area, .7937005+.

ZIELINSKI, AUGUST. Quadrature of the Circle. Augusta, Ga.—Published in *The Analyst*, Vol. II, No. 4, pp. 77-78, July, 1875.

Prof. Zielinski's quadrature is a mechanical one; he says "by means of a single cycloid we can transform any circle into a square."

ADDITIONAL TITLES.

ANDREWS, J. B. The Squared Circle; or the True Area of the Circle Ascertained and Demonstrated. 8vo. Belfast, 1884.

"CANTAB, A." A Hole in Smith's Circle. 8vo. pp. 15. London, 1859.

GLAISHER, J. W. L. An Approximate Numerical Theorem Involving ε (Naperian Base) and π (the Ratio). 8vo. pp. 4. London, 1877.

GLAISHER, J. W. L. Numerical Values of the First Twelve Powers of π, and their Reciprocals, and of certain other related quantities. 8vo. pp. 6. London, 1877.

Five tables are given. I. $\pi, \pi^2, \pi^3, \ldots . \pi^{12}$, to twenty-two or more decimal places. II. $\pi^{-1}, \pi^{-2}, \pi^{-3}, \ldots . \pi^{-12}$ to twenty-two or more decimal places.

LINDEMANN, M. F. Sur le Rapport de la Circonférence an Diamétre, et sur les logarithmes népériens des nombres commensurables ou des irrationnelles algébriques. (10 Juillet, 1882). 4to. pp. 4.

He says, "the number π, or ratio of circumference to diamerer, is a transcendant number."

O'BYRNE, JOHN. An Essay on the Quadrature of the Circle. 8vo. Norfolk, Va.

A two-column review of John A. Parker's work on the "Quadrature of the Circle" appeared in the *Independent Democrat*, Concord, N. H., May 6, 1852, written by W. L. B., of Charlotteville, Va.

Comparison of Ratios, or Values of π.

Names,	Ratio.
ANONYMOUS Benson, Carter, Davies, Ferrel, Fleming, Fisher, Glaisher, Hart, Herschel, Hill, Jackman, Morton, Pierce, Playfair, Rollwyn, Scholfield, Shanks, Upton, Vander-weyde,	3.14159265358 9+
Adorno, Carrick Crabb, Dingle, Gould, L.D., Myers,	$3.142857\frac{1}{7}$
Angherà, DURHAM, Pratt, Rossi, Smooth,	3.2
Davis, QUADRATOR FINALITY,	$3.166666\frac{2}{3}$
Dircks, Merceron, Smith, Jas.,	3.125

Names.	Ratio.
Parker, Skinner, Smith, Seba,	3 1415942+
Baddely,	3.202216+
Bennett,	$3.230769\frac{3}{13}$
Brower,	3.152075+
Bouché,	3.17124864+
B., G. W.,	3.1547+
Cart,	3.1339786+
DeMedici,	$3.14186685\frac{35}{289}$
Drach,	3.14159265+
Faber,	$3.160493827\frac{13}{81}$
FUTURUS,	$3.123809\frac{11}{21}$
Gee,	$3.14159269777454\frac{2}{7}$
Gidney,	3.15135+
Goodsell,	3.1426968+
Chase, Gray,	3.1415+
Grosvenor, Houlston,	3.14213562373+
Harbord,	3.14159265358893- 8193239974916
Harris,	3.142696+
Hornish,	3.15625
Hobbes,	3.1622777+
May,	3.24
Stacy,	3.152965+
Terry,	3.15544362088404- 72216469142611- 31149186928257- 40436092019081- 11572265625
Thompson,	3.174802+
Weatherby,	$3.15333\frac{1}{3}$ (nearly).
Young,	3.174802+
Benson, Rollwyn, Thornton,	Area, $3R^2 = .75$

Andrews, Boyai, Claryvance, Craige, Hudson, Jackson, Murdock, O'Byrne, Steele, Tagen, Thornton, Zielinski,	Ratios not stated.	Dircks,	Supports James Smith.
		Skinner, Smith, Seba,	" J. A. Parker.
		Bennett, Benson, Harbord, Lacomme, Thornton,	The only 5 mentioned by James Smith.
		Baddely, Bennett, Zielinski,	Mechanical quadratures
ANONYMOUS, DURHAM, FINALITY, FUTURUS. QUADRATOR,	Noms de plume.	Angherá, Drach, Hobbes, Houlston, Rossi, Shanks, Smith, Jas.	The only 7 mentioned by A. De Morgan.
Baddely, Faber, Harbord, May, Terry,	$\pi =$ a square.	Hobbes,	$\pi = \sqrt{10}$
		Young,	$\pi = \sqrt[3]{32}$

Augustus De Morgan's Cyclometers.

(From the "Budget of Paradoxes.")

The ratios of these are not stated by De Morgan.

Name.	Year.	Page.	Name.	Year.	B. P. Page.
Beaugrand,	1826	73	JESUIT, So. America,	1844	9
Brauardini, Thomas,	1511	136	Larriva, D. J.,	1856	459
CIVIL ENGINEER,		458	Lausbergii, Philippi	1616	46
Cusa, Nicholas,	1464	33	Macalcarne,	1825	71
De Causans,	1753	179	Philo, of Gadara,		30
De Messange, Mal.,	1686	471	Porta, John Baptiste,	1610	45
De Molina, Cano,		179	Pujos, M.,	1619	179
De Vausenville,	1778	9	Recalcati, Prof.,		458
Dunbar, John,	1619	179	Scaliger, Joseph,	1694	67
Cluvier, Dethleu,	1695	470	Snellii, Willibrordi,	1621	48
Ericius, Nicholas,	1755	97	Sullamar, Henry,	1750	179
Finæus, Orontius,	1544	35	Thompson, T. Perronet,	1856	303
Grange, Armand,		316	Valentinus, Jacobus F.,	1589	36
Gregorio, P.,	1647	71	White, Richard,	1648	9
Gregory, James,	1668	71	Yvon, Paul,		179

Augustus De Morgan's Cyclometers.

Name.	Year.	Ratio.	Page.
Angherà, Domenico,	1854	3.2	289
Peters, William,	1848	3.2	255
Rossi, Gaetano,	1804	3.2	137
Baxter, Thomas,	1732	3.0625	87
Parsey, Arthur,	1832	3.0625	176
Lacomme, Joseph,	1836	3.125	32
Smith, James,	1859	3.125	216
Dean, William,	1863	3.140625	458
JOINER,	1863	3.140625	389
Beaulieu,	1676	3.1622776+	72
Bovillus, Charles,	1503	3.1622776+	31
De Beaulieu, Sieur,	1676	3.1622776+	71
Hobbes, Thomas,	1666	3.1622776+	66
Cataldi, Di Peter A.,	1612	3.14159265358979323846+	46
Gruenberger,	1612	3.14159265358979323846+	
Antiphon,		3.1412+	288
Borello, Pellegrino,		$3.142561983\frac{57}{121}$	46
Bryson,		3.75	288
Campanus,	1503	$3.142857\frac{1}{7}$	31
De Fauré,	1747	$3.160493827\frac{13}{81}$	89
Dennison, Joseph,	1844	$3.177777\frac{7}{9}$	213
De Serres, Olivier,		3.162	288
Drach, Solomon,	1863	3.14159265+	460
Duchesne, Simon,	1558	3.14466+	35
FRIEND, (De Morgan's)		$3\frac{13}{80}+\frac{3}{80}\cos(A-a)$ (A, sun's long.; a, moon's long.)	460
Hailes, John Davey,	1860	3.1424076+	339
Houlston, William,	1862	3.14213562373+	354
Johnson, Henry C.,	1843	$3.048619\frac{1}{21}$	215
Locke, Richard,	1730	3.18897+	87
Longomontanus,	1644	3.14185+	65
McCook,	1841	3.1201937+	214
Metius, Peter,	1640	3.1416017699+	62
Metius, Adrain,			
Phillips, Richard,	1793	3.461016+	145
Smith, Ambrose,	1855	3.1553+	170
Hindu, *Viga Ganita*,		3.1416+	
SPECULATOR,	1842	3.1416+	212
Ptolemy,		3.141552+	[Eng. Cyc. *Art.* "Quadrature."]
Purbach,		3.141667+	" "

Marcelis, Jacob, $3.\frac{1008449087377541679894282184894}{6997183637540819440035139278702}$ 77

Van Ceulen, Lud., 3.14159265358979323846264338327950288 4+ 46

De Morgan was the author of the article on "Quadratures," in the Knight's "English Cyclopædia. He also mentions in his "Budget of Paradoxes," the works of Lipenius, Montucla, and Murhard, who have each written a work on the history of cyclometry.

He also mentions William Rutherford's extended calculations on the value of π, (p 374). William Shanks is credited with carrying the value of π to 607 decimal places (p. 291). Since then Mr. Shanks has extended the decimal to 707 places.

De Morgan gives as an appendix to his "Budget," (pp. 495-500,) Lambert's method of demonstration that no two arithmetical numbers can express the ratio of diameter to circumference. This paper is also given in Brewster's translation of Legendre's Geometry.

There are many methods, entirely independent of the circle, which produce the orthodox value of π. De Morgan gives the following series to infinity:

$$4(1-\tfrac{1}{3}+\tfrac{1}{5}-\tfrac{1}{7}+\tfrac{1}{9}-\tfrac{1}{11}+\tfrac{1}{13}-\tfrac{1}{15}+\ldots) = 3.1415926535897932 3846+$$

The orthodox value of π is usually demonstrated by the method of exhaustions, that is, inscribed and circumscribed polygons. This process has been rigidly criticized by some learned mathematicians. The Legendre Method has been thoroughly examined by J. Ralston Skinner in "The Crown Jewels of the Nations and their Measures," 1877.

The work, "Sources of Measures," by J. Ralston Skinner, Cincinnati, Ohio, 1876, pp. 324, is based on John A. Parker's value of π ($\frac{20612}{6561}$); also, the work "New Elements of Geometry," by Seba Smith, New York, 1850, pp. 200.

The arithmetical harmonies existing among the geometrical forms of triangles, squares, circles, polygons, solids, etc, were made the subject of a paper by the writer, entitled "Squares and Cubes," and read before the Scientific Chapter of the Athens Club of Manchester, N. H., February 12, 1877, especially discussing the "New Law in Geometry," as developed by Theodore Faber of Brooklyn, N. Y., and published in 1865. The paper will be revised and probably published in the near future, making a monograph of abont 24 pages.

Integral Proportions Proposed for the Value of π.

Name.	Proportion.		A Value of π.
Adorno, Juan N.,	7 : 22	: :	1 : $3.142857\frac{1}{7}$
Baddely, William,	361 : 1156	: :	1 : $3.202216\frac{24}{361}$
Baxter, Thomas,	16 : 49	: :	1 : 3.0625
Bennett, John,	26 : 84	: :	1 : $3\ 230769\frac{3}{13}$
Borello, Pelligrino,	484 : 1521	: :	1 : $3.142561983\frac{57}{121}$
Bryson,	4 : 15	: :	1 : 3.75
Cart, Francis G.,	1560 : 4889	: :	1 : $3.133974\frac{14}{39}$
Davis, John,	6 : 19	: :	1 : $3.16666666\frac{2}{3}$
Dean, William,	64 : 201	: :	1 : 3.140625
DeMedici, Charles,	289 : 908	: :	1 : $3.1418685\frac{35}{289}$
De Serres, Oliver,	500 : 1581	: :	1 : 3.162
Dennison, Joseph,	90 : 286	: :	1 : $3.1777777\frac{7}{9}$
Faber, Theodore,	81 : 256	: :	1 : $3.160493827\frac{13}{81}$
FUTURUS,	210 : 656	: :	1 : $3.123809\frac{11}{21}$
Goodsell, S. C.,	1 : $\sqrt{(\frac{800}{81})}$	: :	1 : 3.1426968+
Grosvenor, C. P.,	1 : $\sqrt{(200)}-11$	: :	1 : 3.1421356+
Hindu,	1250 : 3927	: :	1 : 3.1416
Hobbes, Thomas,	1 : $\sqrt{10}$	: :	1 : 3.162776+
Hornish, J. K.,	32 : 101	: :	1 : 3.15625
Johnson, Henry C.,	21 : 64	: :	1 : $3.048619\frac{1}{21}$
Leistner,	1225 : 3844	: :	1 : $3.13795919\frac{18}{49}$
Longomontanus,	43 : $\sqrt{(18252)}$	: :	1 : 3.14185+
May, John,	25 : 81:	: :	1 : 3.24
McCook,	1 : $2+2[\sqrt{(128)}-11]$	: :	1 : 3.1201937+
Metius, Peter,	113 : 355	: :	1 : $3.14159292\frac{3}{113}$
Parker, John A.,	6561 : 20612	: :	1 : $3.1415942\frac{4538}{6561}$
Phillips, Richard,	1 : $\sqrt{12}$	: :	1 : 3.461016+
Rossi, Gaetano,	5 : 16	: :	1 : 3.2
Smith, James,	8 : 25	: :	1 : 3.125
Young, Augustus,	1 : $\sqrt[3]{32}$	: :	1 : 3.1748020+

William Harbord makes the ratio to be a perfect square number, or

$$1 : 1.7724538509054^2 \quad : : \quad 1 : \pi$$

Constant Terry makes the ratio to be a perfect square number, or

$$1 : 1.7763568394002504646778106689453125^2 \quad : : \quad 1 : \pi$$

Theodore Faber makes his area, diameter, and circumference all to be perfect square numbers. His ratio is both a square and a biquadrate: $1\frac{1}{3}$, or $(\frac{4}{3})^2 = \frac{16}{9}$; and $1\frac{7}{9}$, or $(\frac{16}{9})^2 = \frac{256}{81}$, or $3.1604938267\frac{13}{81}$, or

$$1 \;:\; (1.77777777\frac{7}{9})^2 \;::\; 1 \;:\; \pi$$

Diameter $= (1)^2$. Circumference $= (\frac{16}{9})^2$. Area $= (\frac{8}{9})^2$.

Circumscribed square $= (1)^2$. Inscribed square $= (.5\frac{1}{14})^2 - \frac{1}{196}$.

The recurring decimal of Mr. Faber's ratio contains the digits, excepting the digit 5; while his area contains them, excepting the 8.

Thomas P. Stowell has produced from the digits in the form of a common fraction a value of π, now generally in use, as follows:

$$\frac{67389}{21450} = 3.1416.$$

John Bounoulli says that the sum of the following series of fractions, which has unity for numerators and the squares of the natural numbers for denominators, is finite, and equal to the square of the circumference of the circle divided by 6; or the orthodox value of π:

$$\frac{1}{1}+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\frac{1}{25}+\frac{1}{36}+\frac{1}{49}+\frac{1}{64}+\frac{1}{81}, \text{\&c.} = \frac{3.14159265358979323+^2}{6}$$

Wallis, in his *Arithmetic of Infinites*, 1655, gives the following;

$$4 \times \frac{2.4.4.6.6.8.8.10.10.12.12, \text{ etc.}}{3.5.5.7.7.9.9.11.113.13, \text{ etc.}} = 3.14159265358979463238462 6+$$

The Integral Calculus gives the following simple expression in terms of a definite integral:

$$\frac{\pi}{2} = \int_{\infty}^{0} \frac{dx}{1+x^2} = 1.570796326794897661923132 6691+$$

Prof. Benjamin Peirce, in his work, "Linear Associative Algebra," Washington, D. C., 1870, adopted a new symbol for the root of the imaginary quantity $\sqrt{(-1)}$, and produces a result which he terms "the mysterious formula," as follows:

$$\lrcorner = \sqrt{(-1)} \qquad \varepsilon = 2.7182818285+. \qquad \pi = 3.1415926535+$$

$$\lrcorner^{\lrcorner} = \sqrt{\varepsilon^{\pi}} = 4.810477381+.$$

The Solar Equation.

In astronomical works the Greek letter π, the initial of the word *parallaxis*, is also used to represent the solar equation:

$$\pi = 8''.86226925+$$

The parrallactic equation here given is called "the Latimer Solar Equation," from the late Charles Latimer, Cleveland, Ohio, who produces it from the orthodox value of π (the ratio), as follows:

$$\text{Parallactic } \pi = 5\sqrt{\text{Peripheric } \pi}.$$

$$8''.86226925+ = 5\sqrt{3.14159265358+}$$

This value, it will be observed, is ten times the side of a square equal to the area of a circle of diameter one:

$$8''.86226925+ = 10\sqrt{.785398163397448309+}$$

The different calculations of the sun's parallax in modern times as found in works on astronomy, are as follows;

1858	Leverrier,	8″.95	8″.85	8″.86+
1862	Foucault,	.	.	8″.86+
1862	Hall,	.	.	8″.84+
1863	Stone,	.	.	8″.94÷
1863	Hansen,	.	8″.97	8″.91+
1863	Winnecke,	.	.	8″.96+
1864	Powalky,	.	.	8″.86+
1867	Stone,	8″.91	8″.85	8″.916+
1867	Newcomb,	.	.	8″.848+
1868	Stone,	.	.	8″.91+
1868	Faye,	.	8″.70	8″.90+
1871	Powalky,	.	.	8″.7869+
1872	Leverrier,	.	.	8″.86+
1874	Cornu,	.	.	8″.794+
1875	Galle,	.	.	8″.873+
1875	Puiseux,	.	.	8″.879+
1877	Gill,	.	8″.765	8″.815+
1877	Airy,	.	.	8″.754+
1878	Stone,	.	8″.86	8″.979+
1878	Tupham,	.	8″.857	8″.754+

This table gives the results of 29 calculations. The sum of all is 257″.0109+, which divided by 29, gives a mean value for the "solar equation," 8″.862+; thus far it coincides with the Latimer value.

John Taylor, in "The Great Pyramid, Why was it Built?" 1859, says the Pyramid was built for a π - Pyramid. He finds its vertical height is to twice the breadth of its base as diameter to circumference,

$$486.2567 : 763.81 \times 2 : : 1 : 3.1415926535+$$

St. John V. Day, author of "Purpose and Primal Condition of the Great Pyramid of Jeezeh," 1868, computes the area of the Pyramid's right section to the area of the base as 1 to 3.14159265358979+, and adds that it is indeed most singular that the mathematical symbol π (*pi*) for the ratio, is the first letter of the two Greek words *periphreia* and *pyramis*, and intimates that the symbol was probably derived from the latter word because that ratio enters into its several proportions.

Samuel Beswick, in his monograph on "The Sacred Cubit of the Great Pyramid and Solomon's Temple," 1878, says the builders took the circular measure, 3.14159265358979+, and called it a square, and took one side of this square for the first element in the scale of length.

$$\sqrt{3.14859265358979 32}+ = 1.77245385+ \text{ geometrical units.}$$

That the common cubit was ten times these units, or 17.7245385+ geometrical inches; that the royal cubit was 20.6786286+ geometrical inches; that the geometrical inch is = 1.00118+ British inches.

The varied length of some of the cubit-rods is best seen for comparison, as follows:

	Inches.	Feet.
Beswick, Pyramid cubit,	20.7030+	1.7275+
Elephantine,	20.625	1.71875
Harris, from Thebes,	20.650	$1.72083\frac{1}{3}$
Jomard, in Turin Museum,	20.5786+	1.7148+
Jomard, another,	2.06584+	1.7215+
Memphis,	20.4729+	1.706+
Sir Isaac Newton,	20.628	1.719
Nileometer scale,	20.7484+	1.729+
Perring, from Pyramids,	20.6280+	1.7190+
Seyffarth,	20.6111+	1.7175+
Wilkinson, in Turin Museum,	20.5730+	1.7144+

Mr. Beswick says the above approximations of the cubit-rods show they were all intended to represent the same measure, and that their makers had but one standard for a guide.

An interesting illustrated paper on Solomon's Temple by Mr. Beswick will be found in *Scribner's Monthly*, December, 1875, pp. 257-272.